Oil and Gas Pipeline Design, Maintenance and Repair

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Part 3: Single phase Incompressible Flow of Newtonian Fluids

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- Flow regimes
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- Pipe networks unsteady flow in pipes (drainage of reservoirs, water hammers)
- Surge tanks

Overview

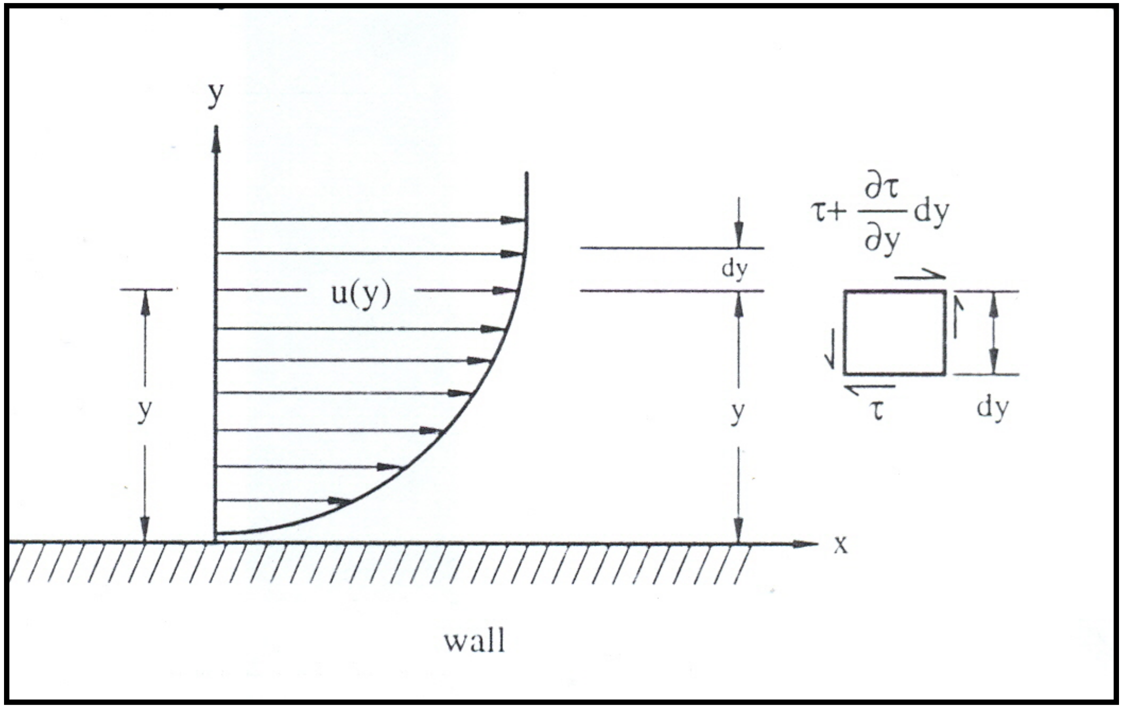
- A **multiphase flow** contains at least two separate phases
- A **single-phase flow,** contains either a single liquid or gas
- **incompressible flow,** the density of any particle in the flow remains constant
- **Homogeneous flow,** the density is constant throughout the flow

Overview (2)

- From elementary fluid mechanics

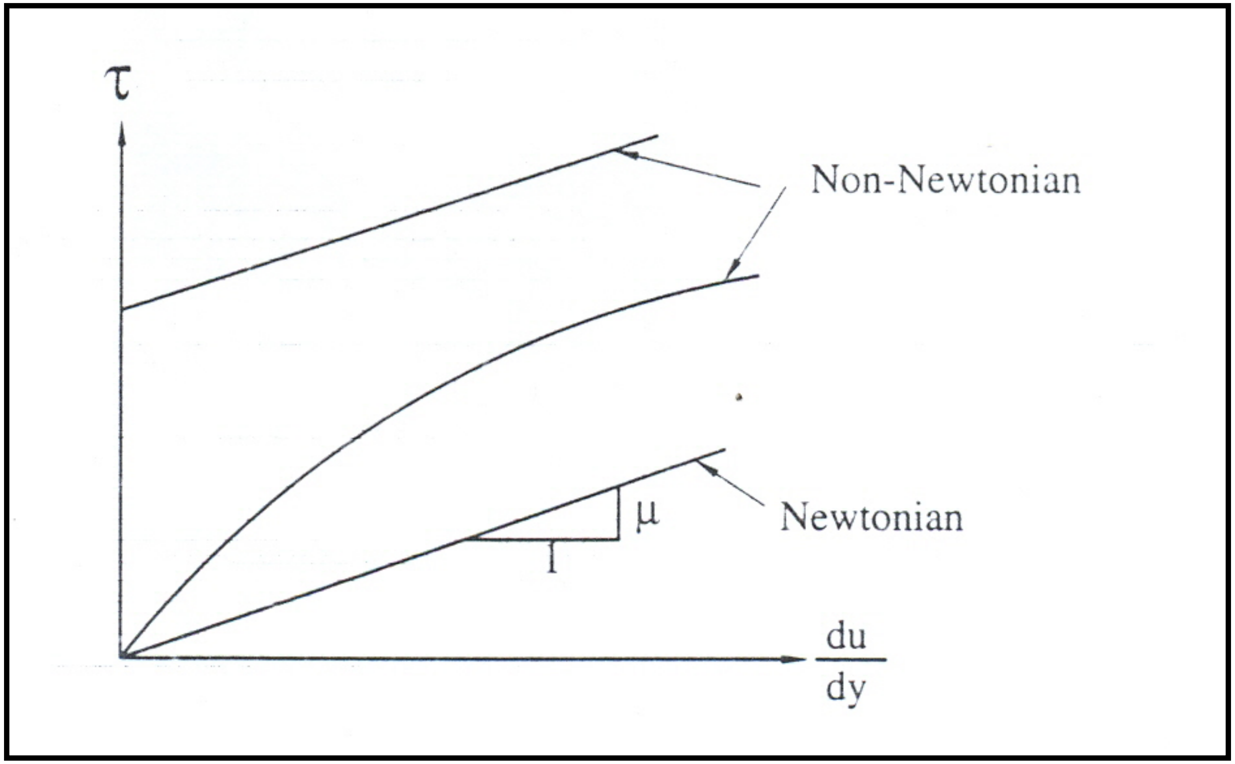
$$\tau = \mu \frac{du}{dy}$$

Overview (3)



Velocity variation from wall and shear stress for a parallel flow

Overview (4)



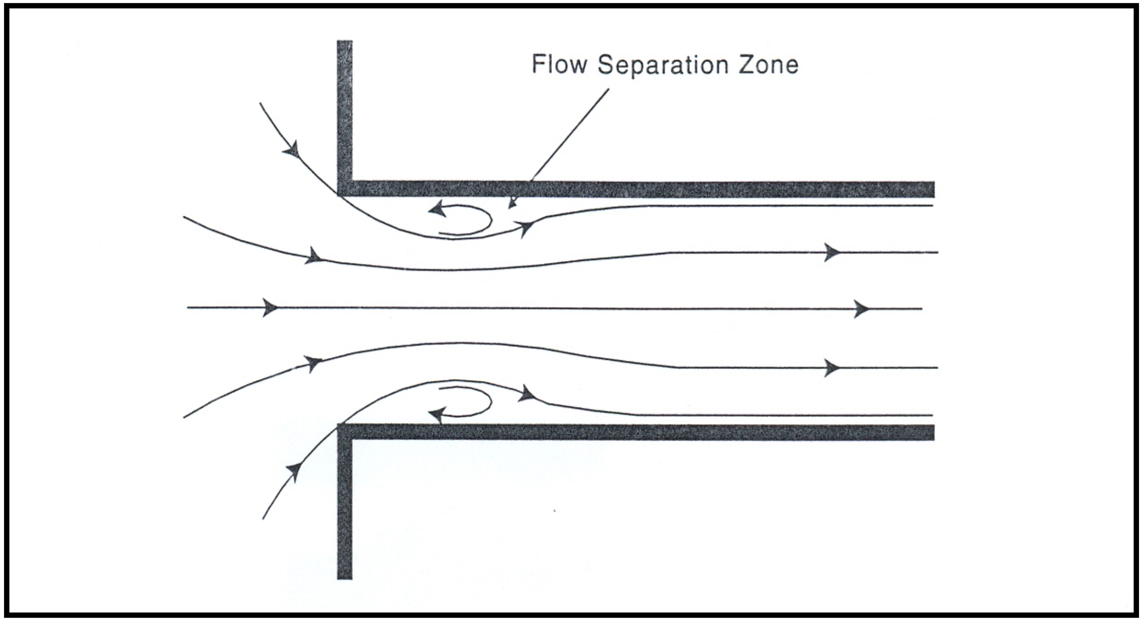
Rheograms of Newtonian and Non-Newtonian

Flow Regimes

- Either laminar or turbulent
- depending on the Reynolds number
- Critical Reynolds is 2100

$$R_N = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

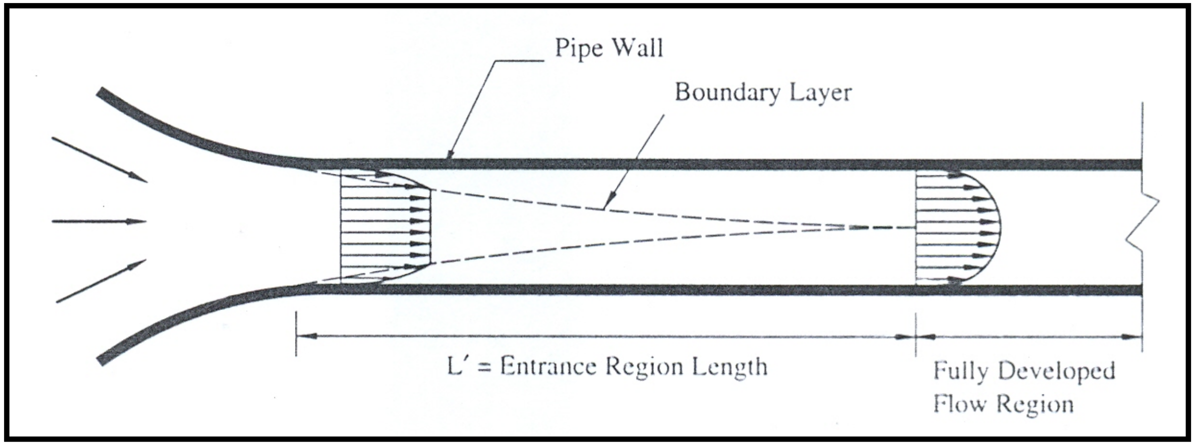
Variation of Velocity along Pipe



Flow pattern near a sharp curved entrance

Variation of Velocity Along Pipe (2)

$$L' = 0.058 D R_N$$



- Concept of fully developed flow in pipe
- *L'* for laminar flow can be calculated from a theoretical formula derived by Langhaar

Example 1

- A man wants to conduct an experiment on laminar flow in a pipe in the fully developed region. He selects a 1-inch-diameter pipe and uses a glycerin/water mixture as the fluid, which has a kinematics viscosity equal to I0-4 ft^2/s. He also selects a mean flow velocity of 2 fps for the test. If he needs a test section of 10 ft of fully developed flow, what should be the minimum length of the pipe used for the experiment?

Solution

- With D = 1 inch = 0.08333 ft, *V* = 2 fps and v = I0-4 ft^2/s,
- The Reynolds number of the flow is R_N = *VD/v* = 1667, which shows that the flow is laminar.
- From Equation 7, the length of the entrance region to establish fully developed flow for this case is [L' = *0.058D* RN = 8.06 ft.
- Therefore, the total length of the pipe needed for the experiment is 8.06 + 10 = 18 ft approximately

Velocity Profile of Fully Developed Flow

$$u = V_c\left(1 - \frac{r^2}{a^2}\right)$$

- For laminar pipe flow, the velocity profile in the fully developed region is a parabola governed by the previous equation
- V_c is the centerline velocity in the pipe; a is the pipe radius; and r is the radial distance.

Example 2

- Determine the relationship between the mean velocity, *u*: in a pipe and the maximum velocity, Vc at the centerline of the pipe, for any fully developed laminar flow

Continuity Equation

$$V_1 A_1 = V_2 A_2 = Q = \text{constant}$$

- A_1 and A_2 are the cross-sectional areas of the pipe at sections 1 and 2
- V_1 and V_2 are the cross-sectional average velocities (mean velocities) at sections 1 and 2
- Q is the discharge

Energy Equation

$$\alpha_1 \frac{V_1^{\,2}}{2g} + \frac{P_1}{\gamma} + z_1 = \alpha_2 \frac{V_2^{\,2}}{2g} + \frac{P_2}{\gamma} + z_2 + h_L + h_t - h_p$$

- subscripts 1 and 2 refer to an upstream section and a downstream section. respectively
- P = average pressure at a cross section;
- z = average elevation of a cross section
- g = the gravitational acceleration
- γ = specific weight of the fluid; a is the energy correction factor to be defined later
- h_L = headloss along the pipe between sections 1 and 2
- h_t = the turbine head
- h_p = the pump head

Energy Equation

- The energy correction factor is

$$\alpha = \frac{\int u^3 ds}{V^3 A}$$

- The integral is to be performed over the entire cross-sectional area.
- α depends on the cross-sectional distribution of the local velocity u.
- For fully developed laminar flow in pipe α = 2.0;
- For fully developed turbulent flow, α is in the neighborhood of 1.05.

Energy Equation

$$\alpha_1 \frac{V_1^{\ 2}}{2g} + \frac{P_1}{\gamma} + z_1 = \alpha_2 \frac{V_2^{\ 2}}{2g} + \frac{P_2}{\gamma} + z_2 + h_L + -h_p$$

- Normally, the term ht in Equation 3.6 is zero because no turbine exists in the pipe between sections 1 and 2.
- Also, α1 and α2 are normally taken to be unity for turbulent flow. Under such conditions

Energy Equation

- Note that the headloss, h_L gives the energy dissipated in the flow by a unit weight of the fluid moving from section 1 to section 2.
- The power lost or dissipated is $$P = \gamma Q h_L$$
- If γ, Q, and hL are in lb/ft^3, cfs, and ft, respectively, then PL is in ft·lb/s.
- In this case, dividing $\gamma Q h_L$ by 550 gives the power dissipated in horsepower

Power Dissipated in A Flow Causes The Temperature

- For a well-insulated pipe (i.e., adiabatic wall), the rise of temperature along the pipe in the flow direction is

$$\Delta T = T_2 - T_1 = \frac{g h_L}{c_v}$$

$$g h_L = c_v (T_2 - T_1) = i_2 - i_1$$

- where c_v is the specific heat capacity at constant volume, and i_1 and i_2 are specific internal energy

Rate of Heat Loss Through The Pipe Wall

$$q = \frac{P_L}{L} = \frac{\gamma Q h_L}{L}$$

$$g h_L = \frac{qL}{\rho Q} = \frac{Q_c}{\rho Q}$$

$$h_L = \frac{c_v \Delta T}{g} + \frac{Q_c}{\rho g Q}$$

Example

- Water is drained by gravity from tank 1 to tank 2 through a steel pipe. The discharge through the pipe is 2 cfs. The difference in the water levels in the two tanks is 100 ft.

1. What is the power dissipated due to headloss?
2. Assuming that the pipe is well insulated, what is the increase in the temperature of the water flowing through the pipe?
3. Assuming that the pipe is not insulated and is ready to conduct heat, what is the rate of the heat loss through the pipe?

Solution

(a) Using Equation 3.6 between two points 1 and 2 at the free surfaces of the two tanks yields

$$\frac{V_1^{\ 2}}{2g}+\frac{P_1}{\gamma}+z_1=\frac{V_2^{\ 2}}{2g}+\frac{P_2}{\gamma}+z_2+h_L+-h_p \qquad \text{(a)}$$

Because V1 = 0. P1 = 0, V2 = 0, and P2= 0, the above equation reduces to

$$z_2\text{-}z_1 = 100 \text{ ft} = h_L \qquad \text{(b)}$$

This shows that regardless of the pipe size and length. the head loss in this case is always the same as the water level difference in the two tanks, which is 100 ft.

Using Equation 3.9, the power dissipated by the flow is

P_L = γQhL,. = 62.4 x 2 x 100 = 12.480 ft-lb/s = 22.7 Hp.

Solution

(b) For the case of a well-insulated pipe, the temperature rise calculated from Equation 3.10 is $\Delta T = 32.2 \times 100/25.050 = 0.13°F$.

This case shows that a rather large headloss results only in a slight increase in the temperature of the water moving through the pipe.

(c)From Equation 3.12, the rate of heat loss through the entire length of the pipe is the same as PL which is 22.7 Hp.

The rate of heat loss per unit length of the pipe is thus q = 22.7/L (in horsepower/ft) where L is the pipe length.

MOMENTUM EQUATION

- The momentum equation for any steady incompressible flow is

$$\vec{F} = \rho \int \vec{V}\vec{V} . d\vec{s}$$

- Where is the resultant force on the control volume; ρ is the fluid density; and is an infinitesimal surface of the control volume.
- Equation 3.15 is written in vector form because it has three components in three mutually perpendicular directions, such as x, y, and z when rectangular coordinates are used

MOMENTUM EQUATION

- Application of Equation 3.15 to an incompressible flow between two sections, 1 and 2, of a straight pipe yields

$$F = \rho\left(\beta_2 V_2^{\,2} A_2 - \beta_1 V_1^{\,2} A_1\right) = \rho Q\left(\beta_2 V_2 - \beta_1 V_1\right)$$

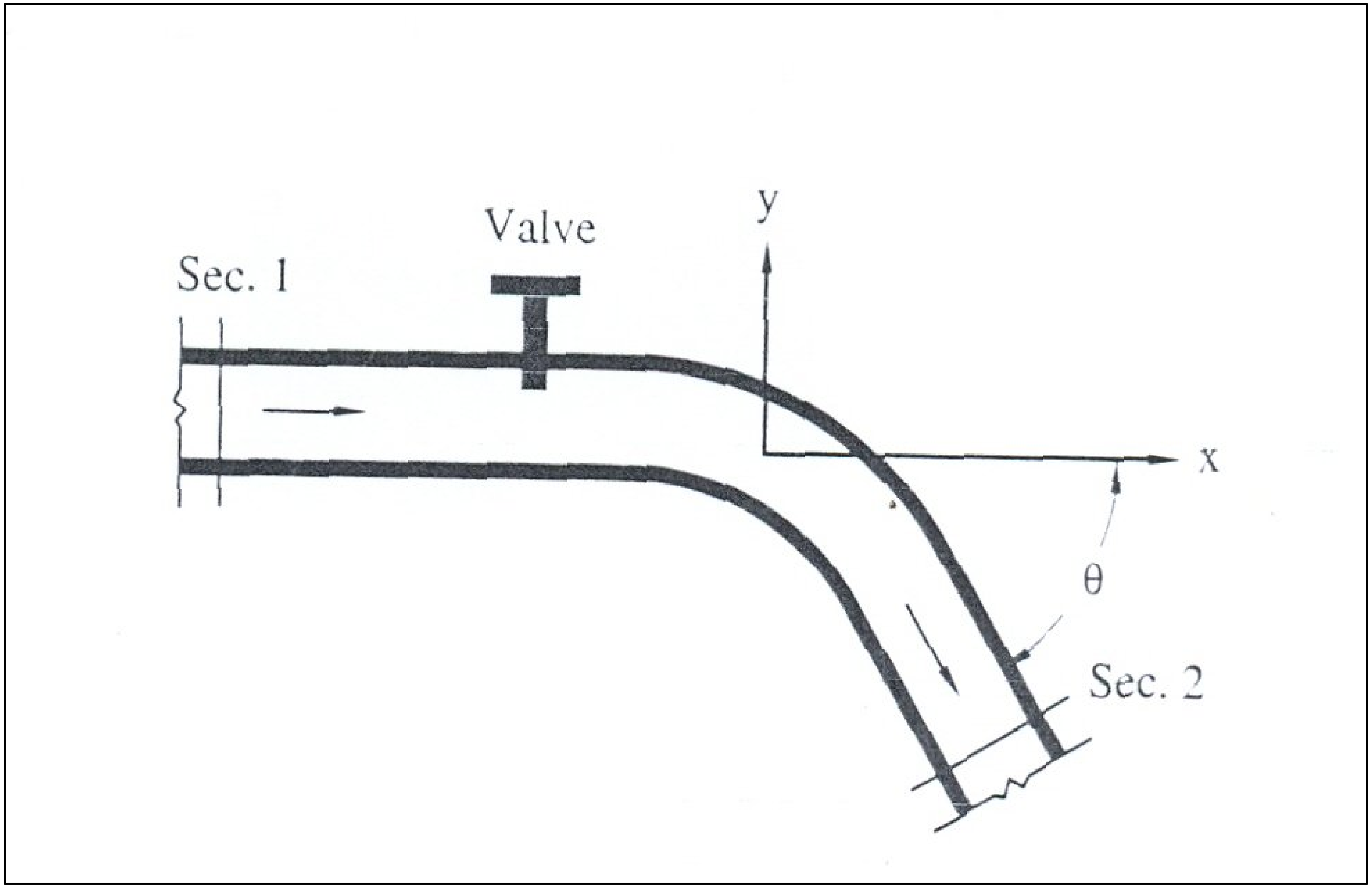
- F = force on the fluid in the flow direction
- A = cross-sectional area
- V = cross-sectional mean velocity
- Q = discharge
- β = momentum correction factor
- subscripts 1 and 2 refer to an upstream and a downstream cross section

Momentum Correction Factor

$$\beta = \frac{\int u^2 ds}{V^2 A}$$

- For fully developed laminar and turbulent flows in pipes, the values of β are respectively 4/3 and 1.02, approximately.
- In ordinary practice, especially in ordinary turbulent pipe flow, β is taken to be 1.0, and so β 1 and β 2 in Equation 3.16 disappear from the equation.

Analysis of Fluid Forces Acting on a Pipe Bend



Analysis Of Fluid Forces Acting on a Pipe Bend

$$F_x = \rho\left(\beta_2 V_2^{\,2} A_2 \cos\theta - \beta_1 V_1^{\,2} A_1\right)$$

$$= \rho Q\left(\beta_2 V_2 \cos\theta - \beta_1 V_1\right)$$

$$F_y = -\rho \beta_2 V_2^{\,2} A_2 \sin\theta$$

$$= -\rho \beta_2 Q V_2 \sin\theta$$

HEADLOSS FORMULAS

- The headloss term, h_L in Energy Equations includes both the loss distributed uniformly along a pipe - the so-called major loss - and the loss due to localized disturbances - the so-called minor losses.
- The terms major loss and minor losses are misnomers because for a short pipe containing fittings that disturb the flow, such as valves or bends, the minor losses can be much greater than the major loss.

Fitting Loss

$$h_L = K \frac{V^2}{2g}$$

- K = local loss coefficient, which is approximately constant for a given fitting as long as the flow is turbulent

Fitting Loss

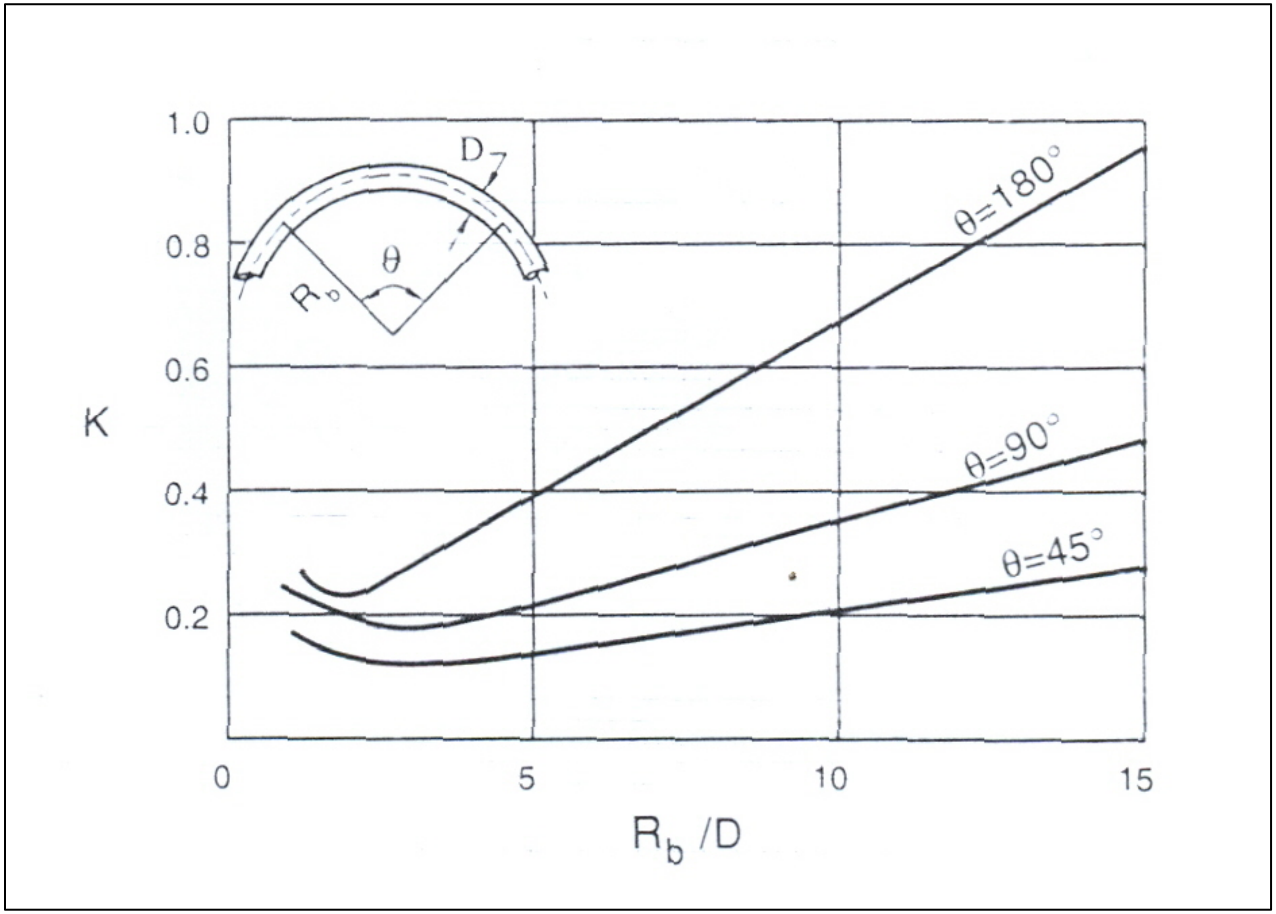
$$h_L = f \frac{L_e}{D} \frac{V^2}{2g}$$

- L_e = Equivalent pipe length,
- f = Darcy- Weisbach friction factor

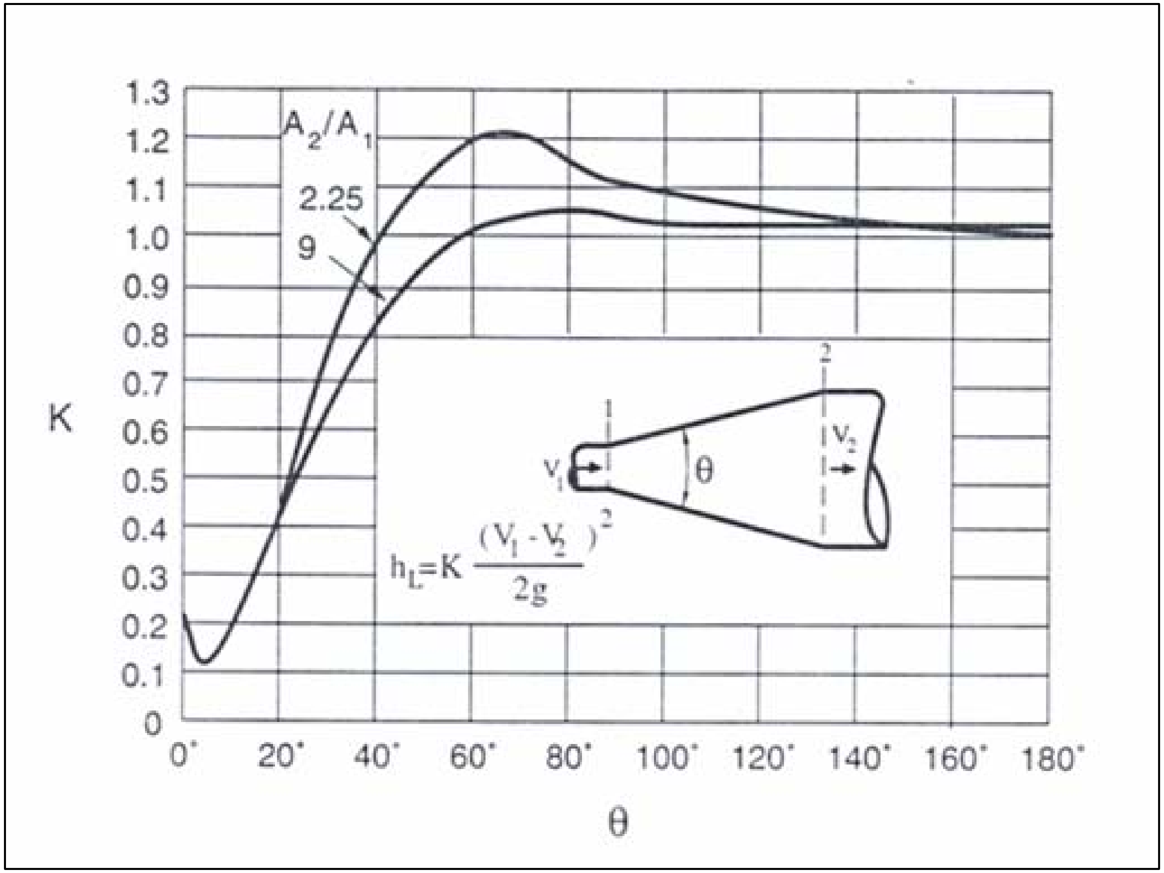
Typical Headloss Coefficient K For Selected Fittings

Fitting	K Value	Fitting	K Value
Check valves		**Standard T**	
B all type	70	Side outlet	1.8
Disc type	10	Straight-through flow	0.4
Swing type	2		
Other valves		**Elbows (90°)**	
Foot valve	10	Regular	1.0
Globe valve	8	Long radius	0.4
Angle valve	3	Elbows (45°)	
Diaphragm valve	2	Regular	0.3
Gate valve	1.5	Long radius	0.2
Butterfly valve	0.2		
Full-bore ball valve	Negligible (<0.1)	Return bend	2.2

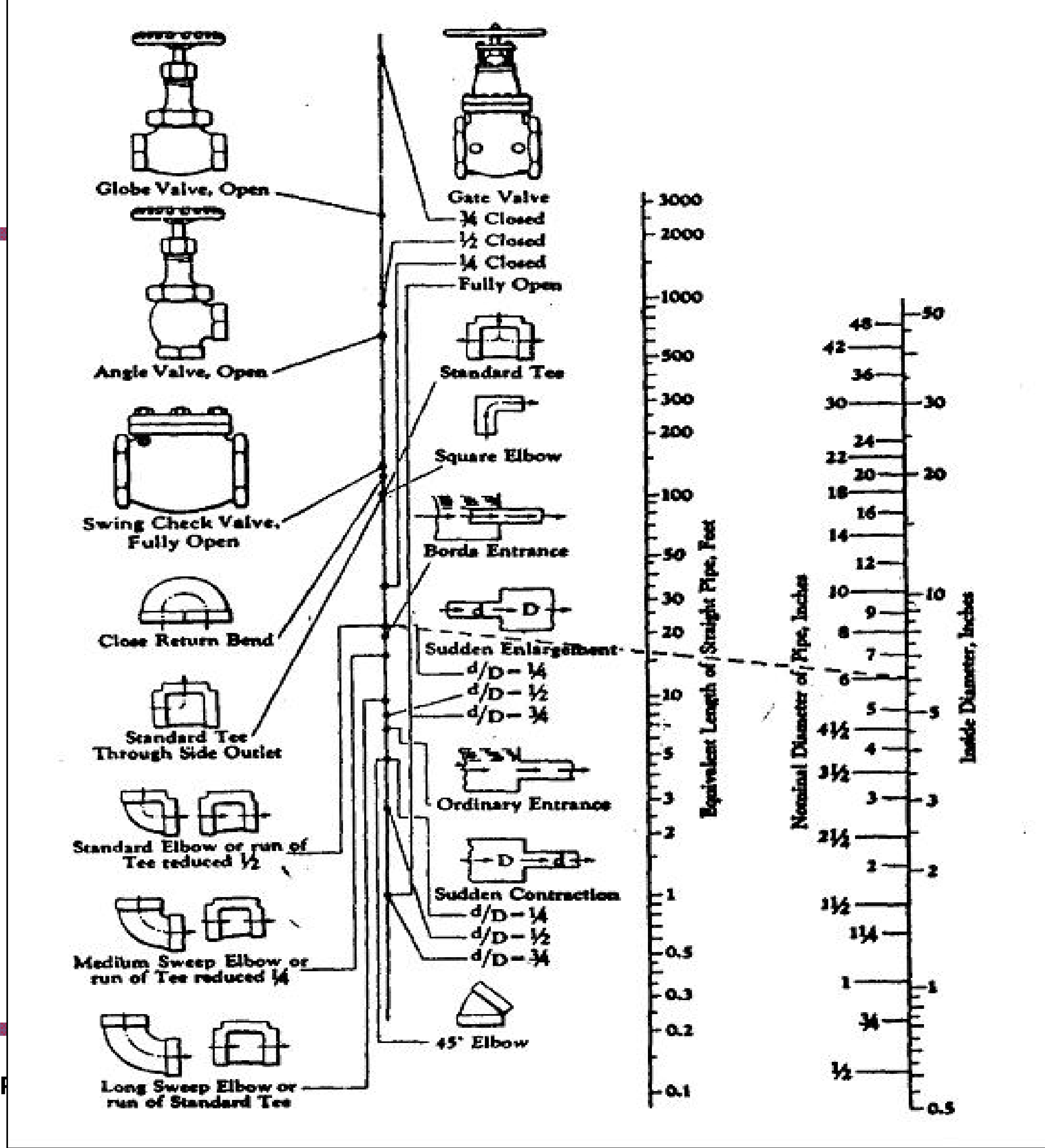
Headloss Coefficient K for a Smooth Pipe Bends (R_n =2x105)



Headloss Coefficient K for a Conical Enlargement (Diffuser)



Monogram for Equivalent Length



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Pipe Loss (Major Loss)

- Darcy- Weisbach formula

$$h_L = f \frac{L}{D}\frac{V^2}{2g}$$

- For Laminar flow

$$f = \frac{64}{R_N}$$

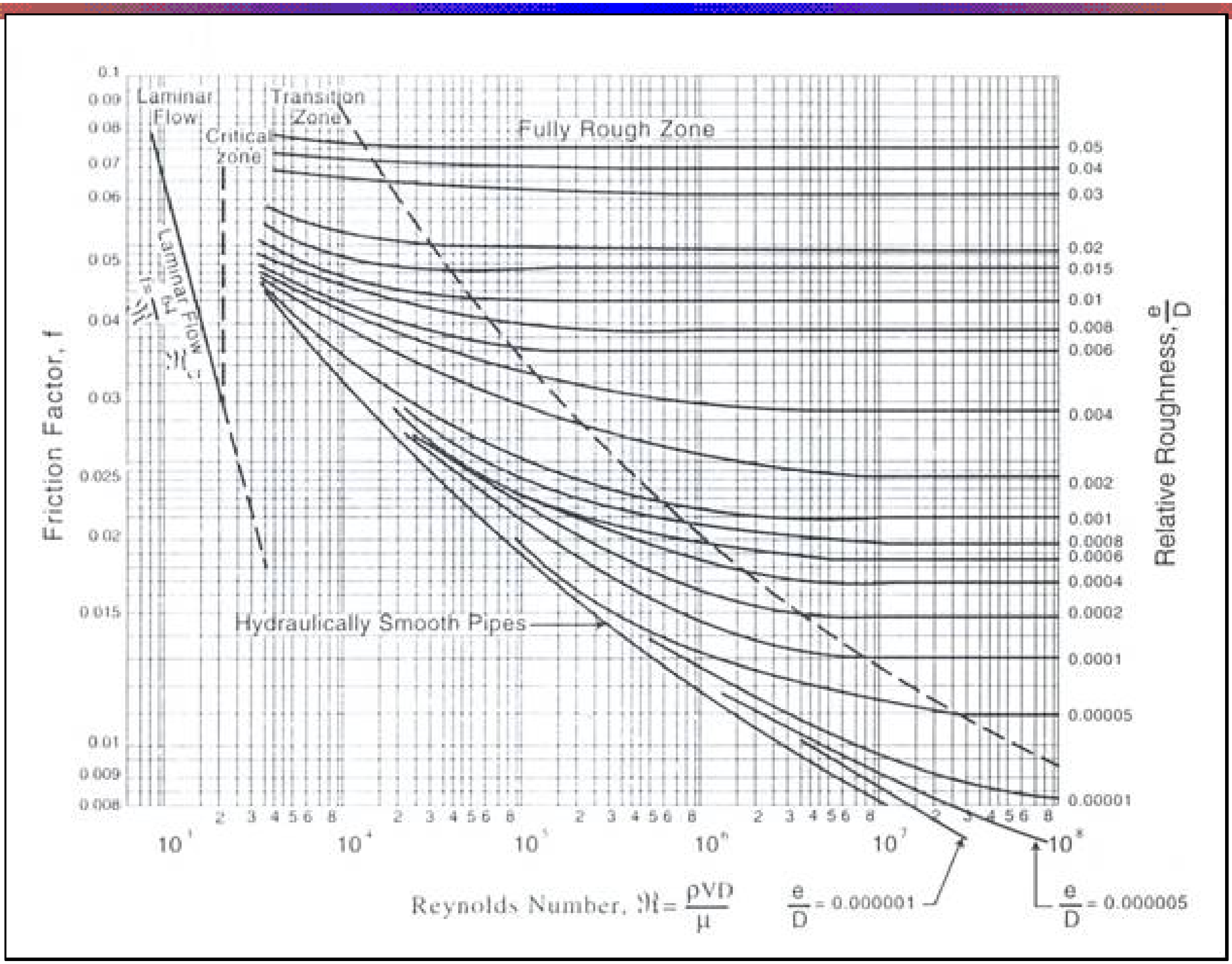
- For Turblent flow, Colebrook formula

- $$\frac{1}{\sqrt{f}} = -0.86\ell n\left(\frac{e/D}{3.7} + \frac{2.51}{R_e\sqrt{f}}\right)$$ or $$\frac{1}{\sqrt{f}} = 1.14 - 2.0\log\left(\frac{e}{D} + \frac{9.35}{R_N\sqrt{f}}\right)$$

Absolute Roughness

Pipeline Material	Absolute Roughness, e	
	ft	mm
Glass and various plastics (e.g., PYC and PE pipes)	0 (hydraulically smooth)	0 (hydraulically smooth)
Drawn tubings (e.g., copper or aluminum pipes or tubings)	$5x10^{-6}$	$1.5x10^{-6}$
Commercial steel or wrought iron	$1.5x10^{-4}$	$4.6x10^{-2}$
Cast iron with asphalt lining	$4x10^{-4}$	0.12
Galvanized iron	$5x10^{-4}$	0.15
Cast iron	$8.5x10^{-4}$	0.25
Wood stave	$6x10^{-4}$ – $3x10^{-3}$	0.18-0.9
Concrete	$1x10^{-3}$ - $1x10^{-2}$	0.3-3.0
Riveted steel	$3x10^{-4}$ - $3x10^{-4}$	0.9-9.0

Moody Diagram



Other Friction Factors Formulas

- Haaland $$\frac{1}{\sqrt{f}} = -0.782\ell n\left(\left(\frac{e}{3.7D}\right)^{1.11} + \frac{6.9}{R_N}\right)$$
- Blasius $$f = \frac{0.316}{R_N^{1/4}}$$
- completely rough zone of turbulent flow

$$f = \frac{1}{\left[1.14 - 2.0\log(e/D)\right]^2}$$

Other Form of Darcy-Weisbach Formula (Fanning's Equation)

$$h_L = f' \frac{L}{R_H} \frac{V^2}{2g}$$

- *f'* is the Fanning's resistance factor
- R_H is the hydraulic radius, which is the area A divided by the wetted perimeter p.
- For a circular cross-sectional pipe flowing full, RH = D/4.
- Therefore, *f* = 4*f'*.

Hazen-Williams Formula

$$V = 0.55 C_H D^{0.63} S_e^{0.54}$$

- C_H is the Hazen-Williams coefficient and S_e is the energy slope, which is the head loss per unit length of pipe, h_L/L
- Ordinarily, engineers consider C_H to be a function of pipe materials only, having a constant value for any given pipe material.
- For smooth pipe interior, the value of CH used is 140; C_H decreases as the pipe gets rougher.
- With severely tuber calculated old water pipes values of C_H less than 80 may be used.

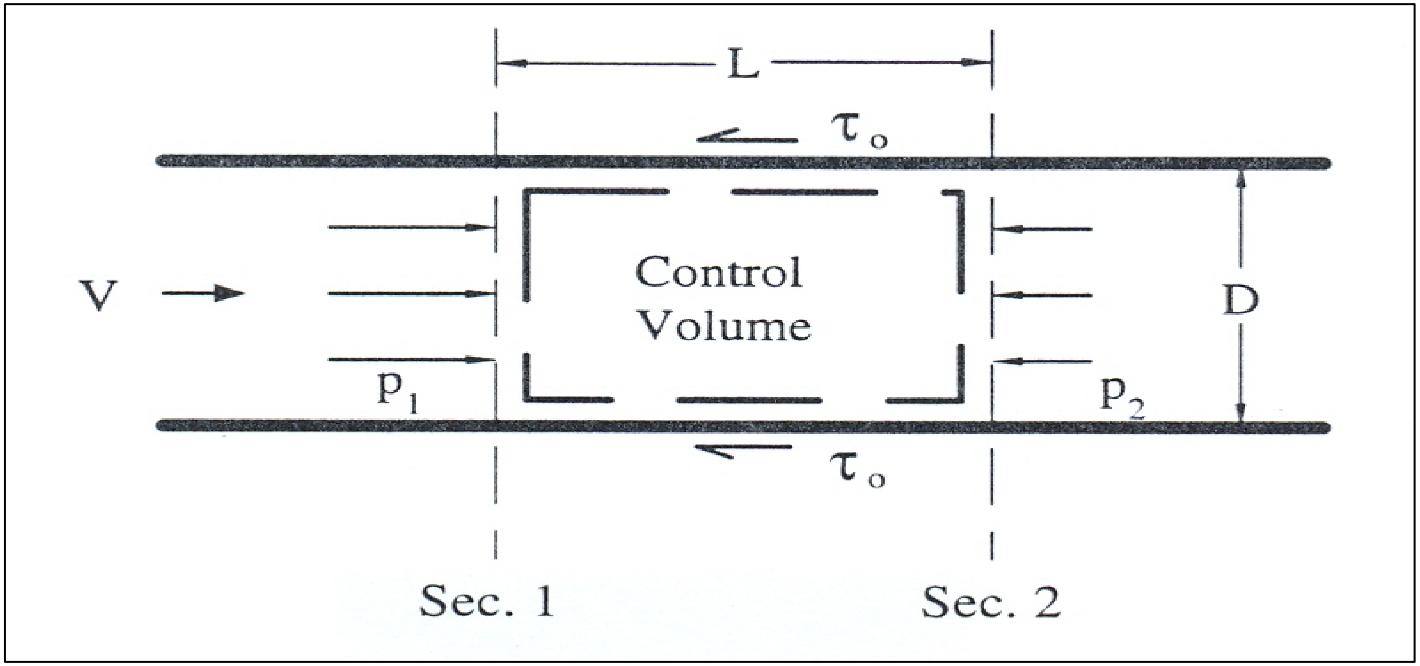
Manning Formula

$$V = \frac{1.486}{n} R_H^{2/3} S_e^{1/2}$$

- n is the Manning's roughness coefficient, which is approximately equal to o.14 for an unpolished concrete surface

Total Loss

$$h_L = \sum K \frac{V^2}{2g} + f \frac{L}{D}\frac{V^2}{2g} = \frac{V^2}{2g}\left(\sum K + f \frac{L}{D}\right)$$



Force balance analysis to determine the shear stress on pipe wall From the energy equation

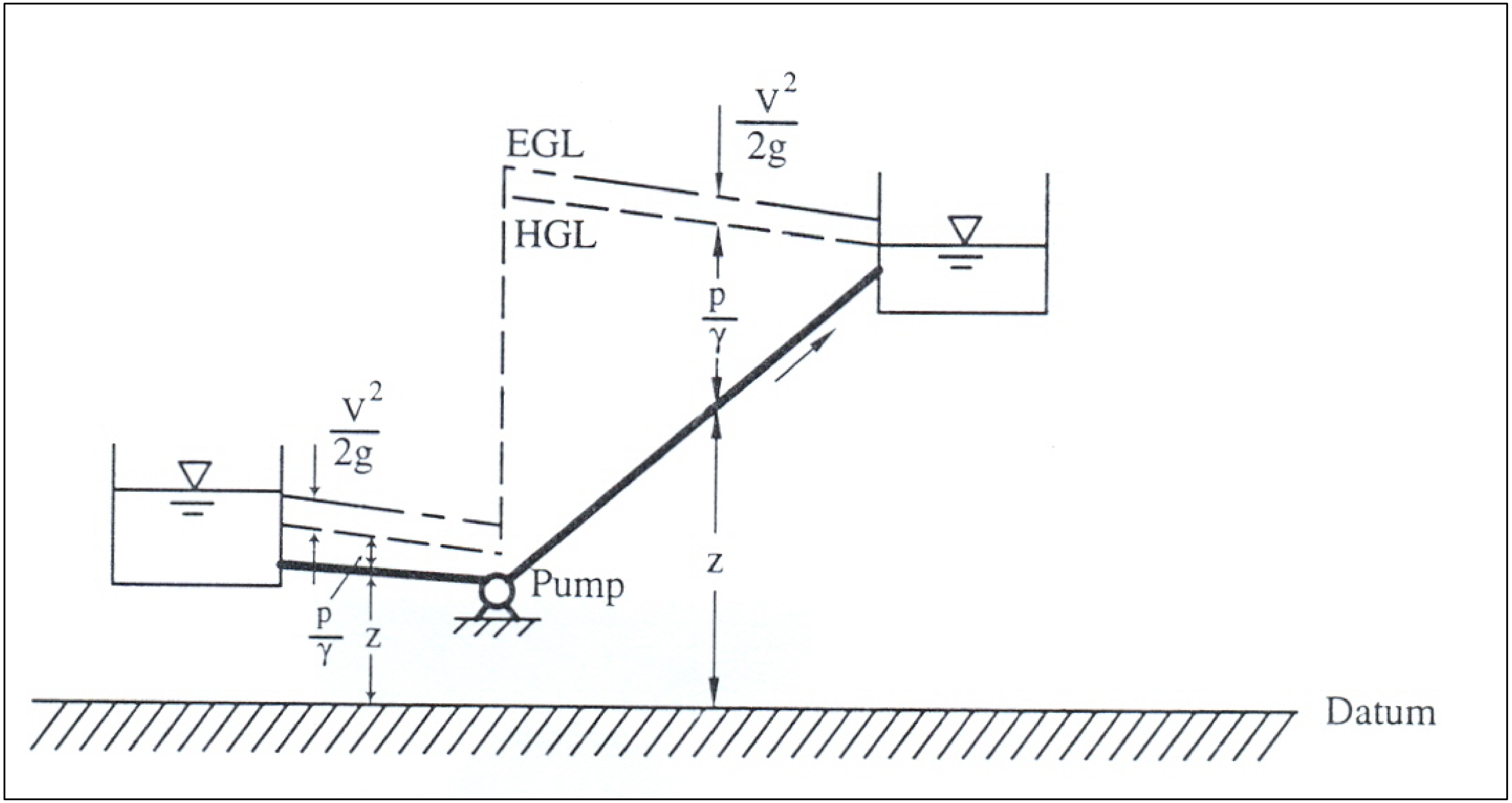
SHEAR ON PIPE WALL

$$\frac{P_1 - P_2}{\gamma} = h_L = f\ \frac{L}{D}\frac{V^2}{2g}$$

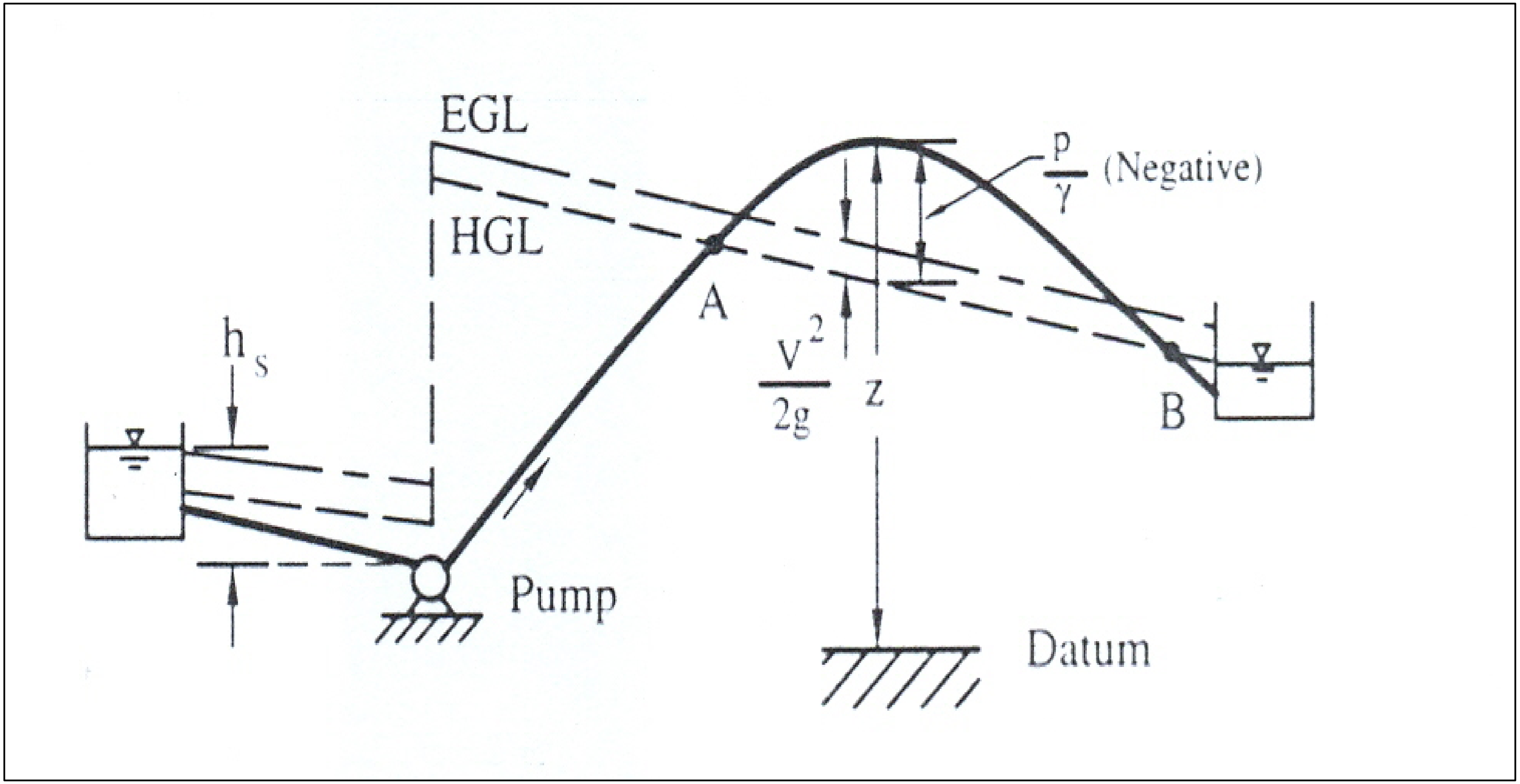
$$F = \left(P_1 - P_2\right)A - \tau_0 \pi D L = 0$$

$$\tau_0 = \frac{(P_1 - P_2)A}{\pi D L} = \frac{(P_1 - P_2)D}{4L}$$

HYDRAULIC AND ENERGY GRADE LINES



HYDRAULIC AND ENERGY GRADE LINES



Cavitation in Pipeline Systems

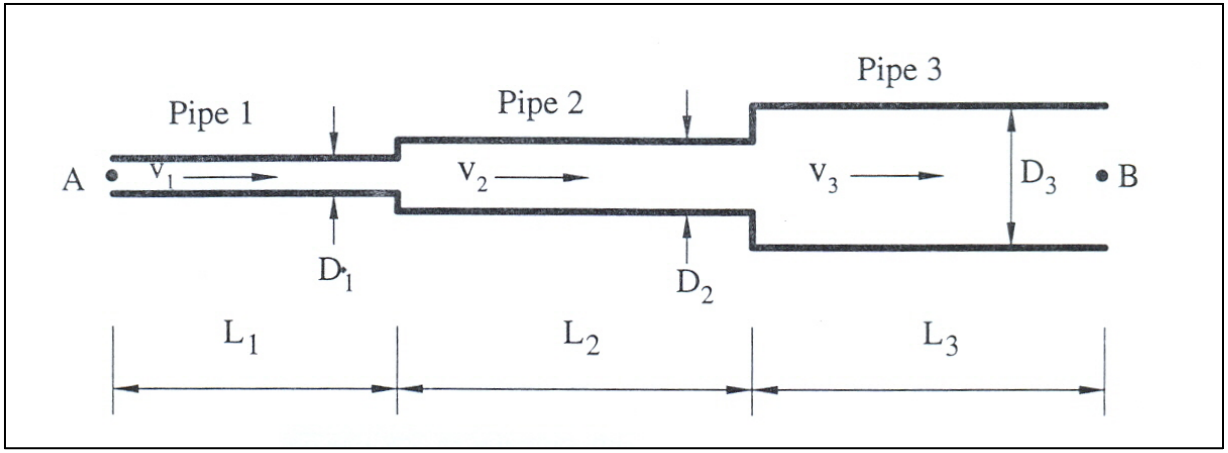
- Whenever the pressure of the liquid flowing in a pipe drops to vapor pressure, P_v, the liquid evaporated or boils.
- The phenomenon is called cavitations
- It interrupts the flow
- Cavities causes damage to the interior of the pipe and its fittings, and to pump and turbine blades or vanes
- Cavitations may take place in a region of low pressure (or high suction).
- To prevent, the designer must calculate the available NPSH (net positive suction head) and compare it with the required NPSH for the pump

Net Positive Suction Head (NPSH)

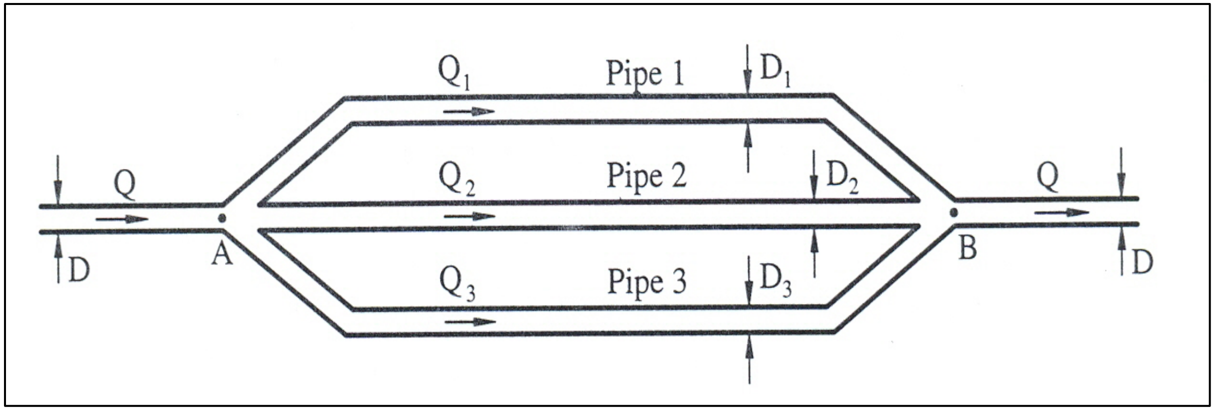
- NPSH (available) = $h_a + h_s - h_L - h_v$
- h_a is the absolute pressure head of the atmosphere (approximately 34 ft or 10.4 m of water under standard atmospheric conditions)
- h_s is the static head, which is the height of the liquid in the intake reservoir above the pump elevation
- h_L is the head loss in the pipe from the pipe entrance to the suction side of the pump, including both pipe loss and local losses
- h_v is the vapor pressure head of the liquid, p_v/γ, which is a function of temperature

Pipes in Series and Parallel

Series Pipelines



Parallel Pipelines



Pipes in Series

$$\left(h_L\right)_{AB} = f_1 \frac{L_1}{D_1}\frac{V_1^{\,2}}{2g} + f_1 \frac{L_1}{D_1}\frac{V_1^{\,2}}{2g} + f_1 \frac{L_1}{D_1}\frac{V_1^{\,2}}{2g} + \frac{(V_1 - V_2)^2}{2g} + \frac{(V_2 - V_3)^2}{2g}$$

$$Q = V_1 A_1 = V_2 A_2 = V_3 A_3$$

Pipes in Parallel

- $Q_1 + Q_2 + Q_3 = Q$

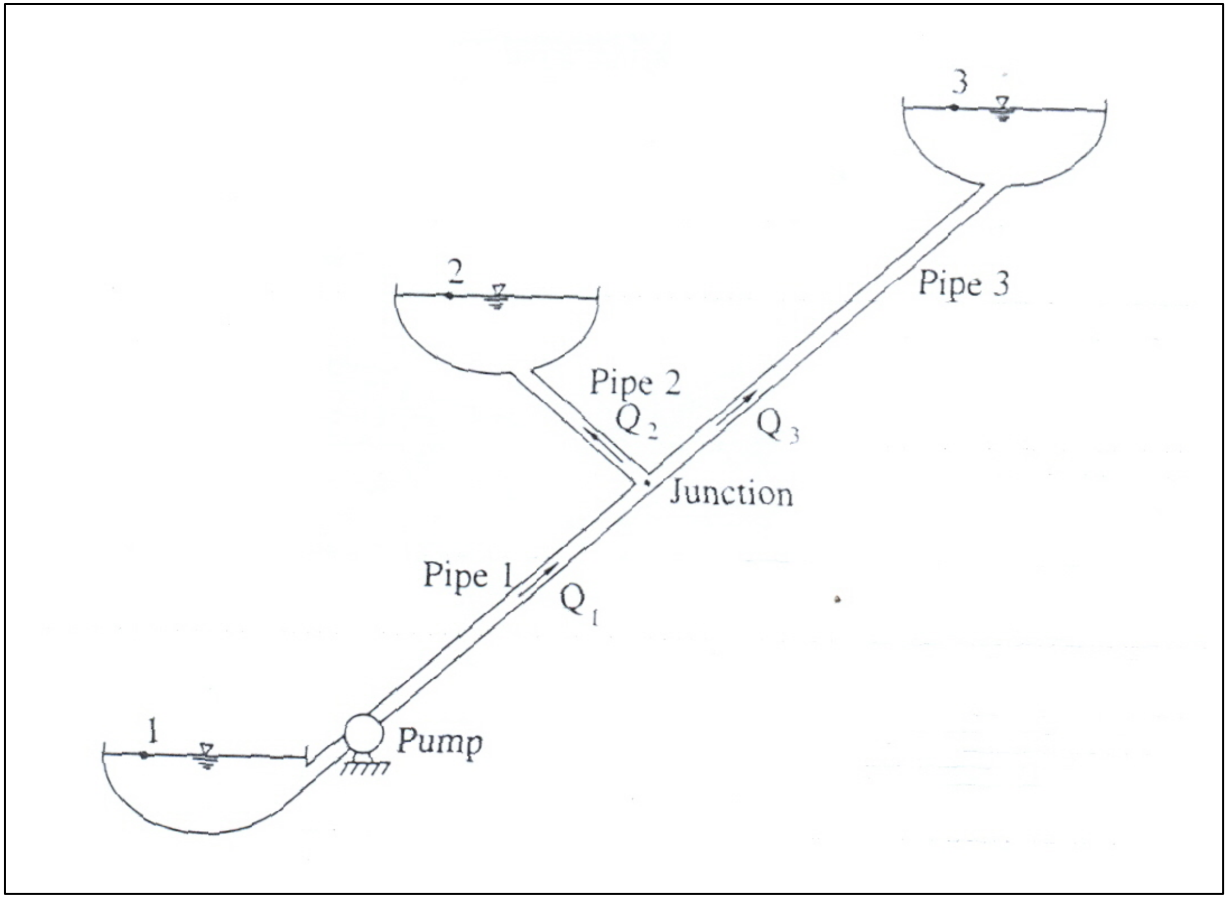
$$V_1 = \frac{Q_1}{A_1}, V_2 = \frac{Q_2}{A_2}, V_3 = \frac{Q_3}{A_3}$$

$$h_{L1} = f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} + \sum K_1 \frac{V_1^2}{2g}$$

$$h_{L2} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} + \sum K_2 \frac{V_2^2}{2g}$$

$$h_{L3} = f_3 \frac{L_3}{D_3} \frac{V_3^2}{2g} + \sum K_3 \frac{V_3^2}{2g}$$

Interconnected Reservoirs



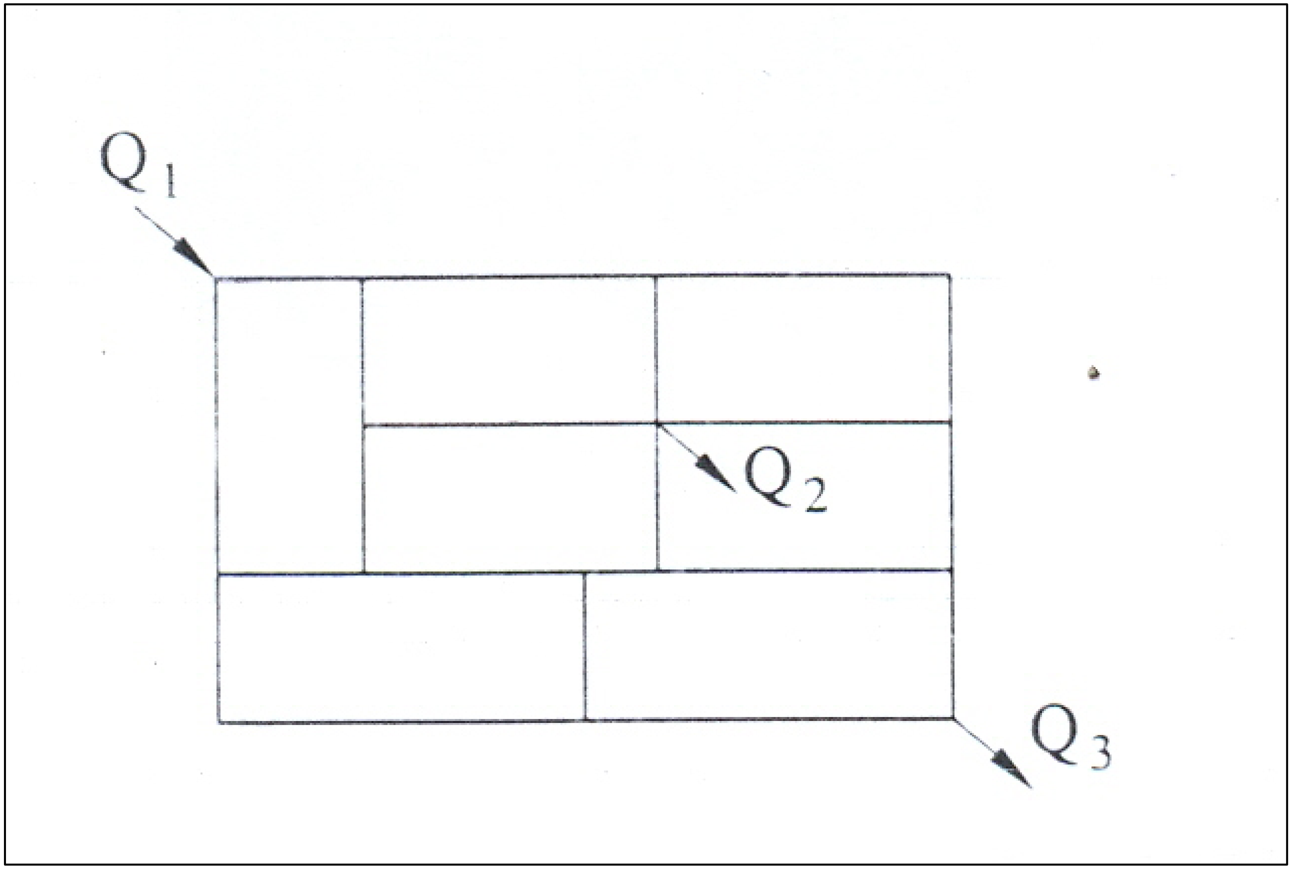
Solved Using The Following Steps

- Assume a discharge Q1, through the pump and through Pipe 1.
- Use the energy equation between reservoir 1 and the junction of the pipes.
- Find the piezometric head at the junction, h_j
- Note that the pump head in the energy equation can be determined from the pump characteristic curve, which gives the pump head as a function of discharge.
- Compare the piezometric head at the junction with the water surface elevation of reservoirs 2 and 3.

Solved Using The Following Steps

- If the piezometric head at the junction is higher than the height of water in any reservoir, the flow goes from the junction to the reservoir. Otherwise, the flow will be in the reverse direction.
- Using energy equation between the pipe junction and each reservoir, the velocity and the discharge in pipes 2 and 3 can be calculated.
- Check if continuity equation ($Q_1 = Q_2 + Q_3$) is satisfied at the junction. If not, a new discharge must be assumed in pipe 1 and the same six steps followed until $Q_1 = Q_2 + Q_3$
- Check the required and available NPSH to make sure that the pump does not cavitate.

Pipe Network



Pipe Network

- The algebraic sum of the pressure drops around each circuit (loop) in the network is zero. namely,
- $\sum hL = 0$ (around each loop)
- Continuity equation must be satisfied at each junction (node). This means all flows going into a junction must equal to flows leaving the junction, or there is no net inflow or outflow at each junction or node, namely,
- $\sum Q = 0$ (at each loop)
- Headloss across each pipe can be determined from the Darcy-Weisbach equation.
- Fitting losses for each pipe can be neglected when valves in the pipe are fully open and when the pipe is relatively long-say at least 500 times the pipe diameter.

Pipe Network

$$h_L = C_0 Q^2$$

$$C_0 = \frac{8fL}{g\pi^2 D^5}$$

$$\sum h_L = \sum C_0 Q^2 = 0$$

Pipe Network

$$\Delta Q = \frac{\left(\sum_{c} h_L - \sum_{cc} h_L\right)}{2\left(\sum_{c} \frac{h_L}{Q} - \sum_{cc} \frac{h_L}{Q}\right)}$$

$$\Delta Q = \frac{\left(\sum_{c} C_0 Q^2 - \sum_{cc} C_0 Q^2\right)}{2\left(\sum_{c} C_0 Q - \sum_{cc} C_0 Q\right)}$$

Unsteady Flow in Pipe

- When the discharge or velocity of the flow in a pipe varies with time, the flow is said to be unsteady.
- Two types of unsteady flow are treated herein, those that vary slowly with time and those that vary rapidly with time.
- The former, the quasi-steady flow, can be treated with negligible error by using steady incompressible flow solution at any given time.
- The latter must be treated as truly unsteady state flow with due regard to compressibility effect.

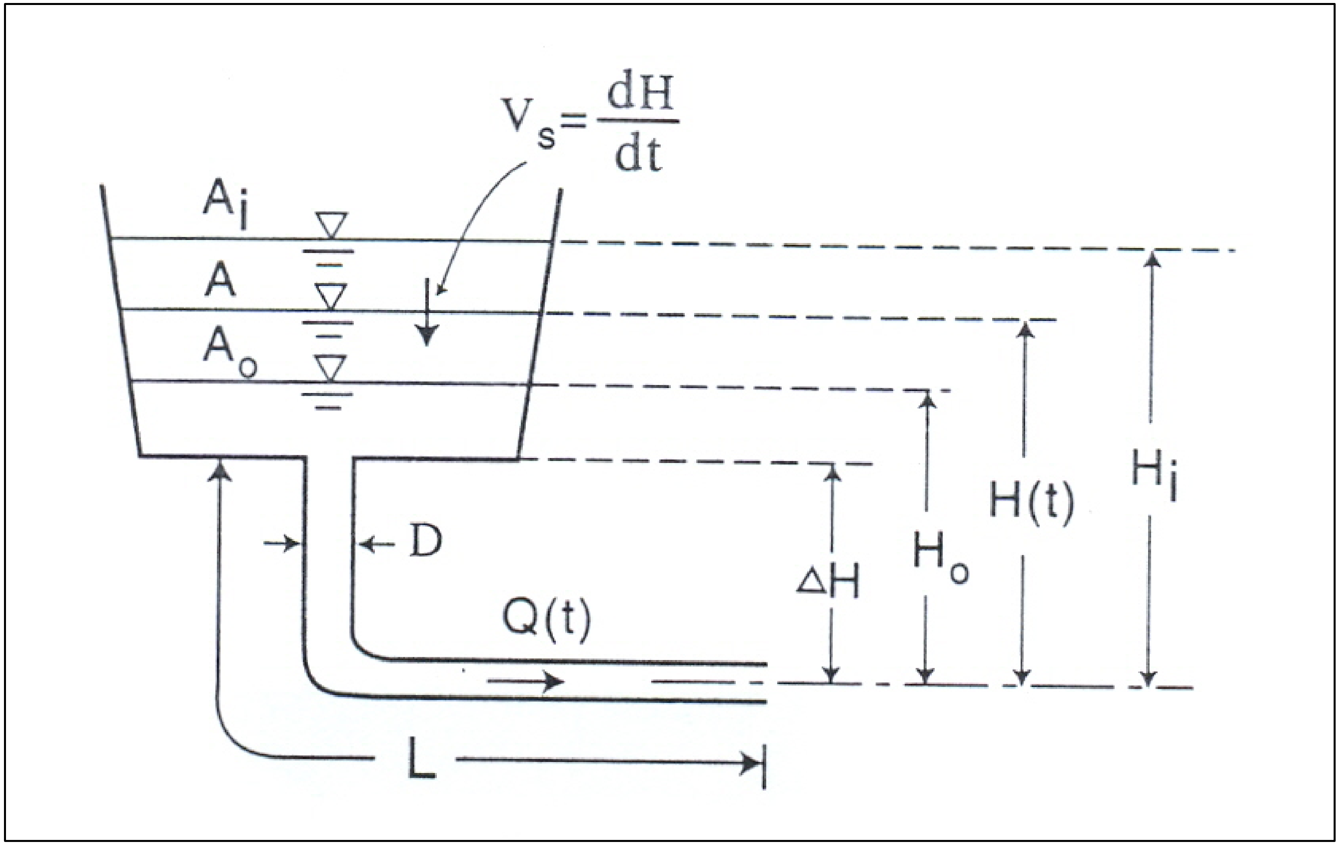
Quasi-steady Solution

They include

- Drainage of a reservoir or pipe
- Fluid acceleration due to valve opening
- Flow oscillations in interconnected reservoirs

Drainage of A Reservoir or Pipe

Pipeline drainage of liquid from a reservoir



Drainage of A Reservoir or Pipe

- Steady-state energy equation at any given time t, when the water height is H.
- Using Equation between the water surface (point 1) and the pipe outlet (point 2) yields

$$H=\left(1+\sum K+f\frac{L}{D}\right)\frac{V^2}{2g} \qquad V=\sqrt{\frac{2gH}{1+\sum K+f\dfrac{L}{D}}}$$

$$Q=VA=-A_s\frac{dH}{dt}$$

Drainage of A Reservoir or Pipe

- Combination the equations yields

$$\frac{A_s dH}{\sqrt{H}} = -A\sqrt{\frac{2g}{1+\sum K + f\,\frac{L}{D}}}dt$$

- Integrating the above equation and using the conditions that H = H_i, when t = 0 and H = H_0 when t = t_0, the following result is obtained

$$t_0 = \frac{\sqrt{1+\sum K + f\,\frac{L}{D}}}{A\sqrt{2g}} \int_{H_0}^{H_i} \frac{A_s}{\sqrt{H}} dH$$

Drainage of A Reservoir or Pipe

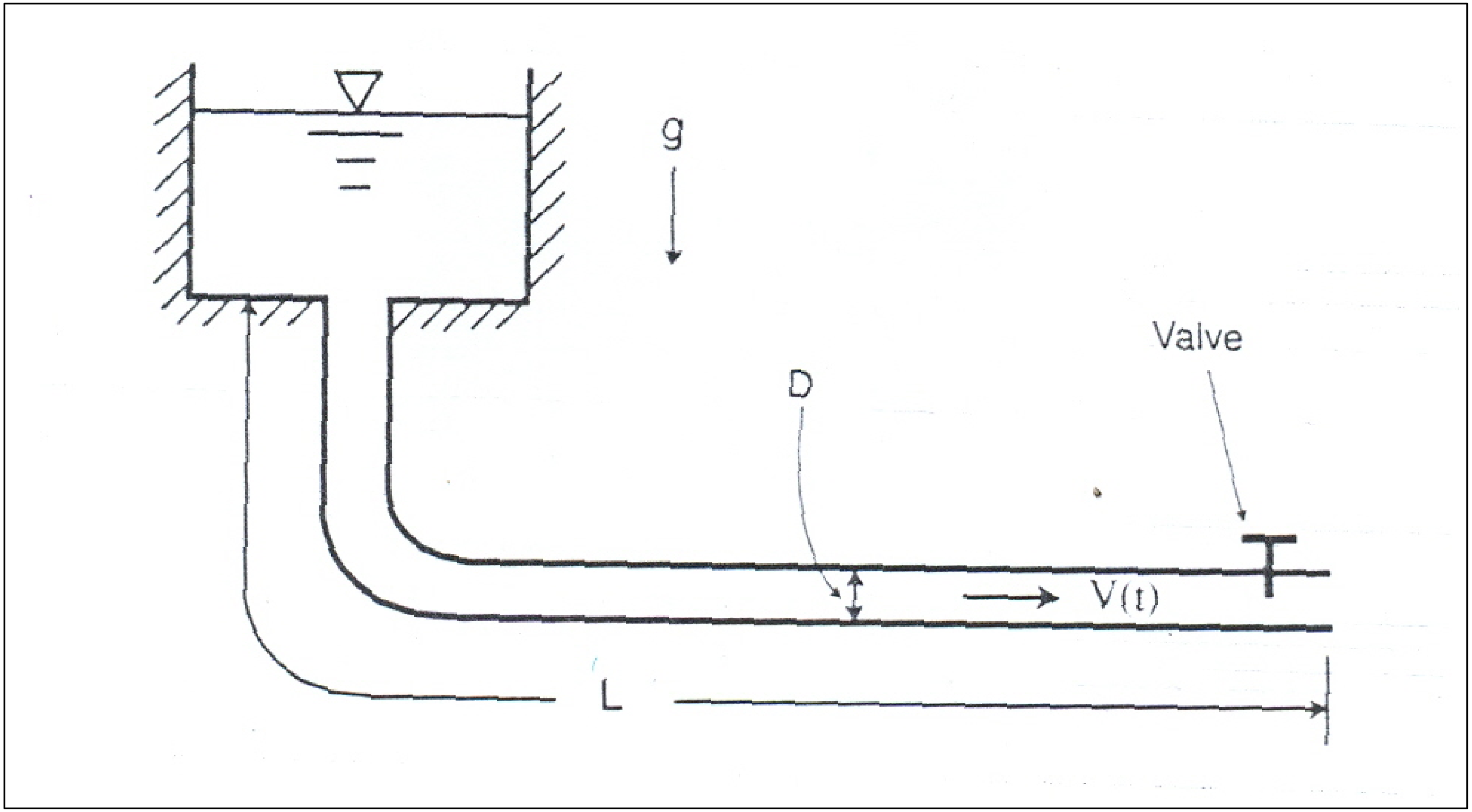
- if A, does not vary

$$t_0 = \frac{\sqrt{1 + \sum K + f\ \frac{L}{D}}}{A\sqrt{2g}} \left(H_i^{1/2} - H_0^{1/2} \right)$$

- Another similar example is to drain a nonhorizontal pipe by gravity

$$t_0 = \frac{f^{1/2} L^{3/2}}{A\sqrt{2gD\Delta H}}$$

Fluid Acceleration Due to Valve Opening



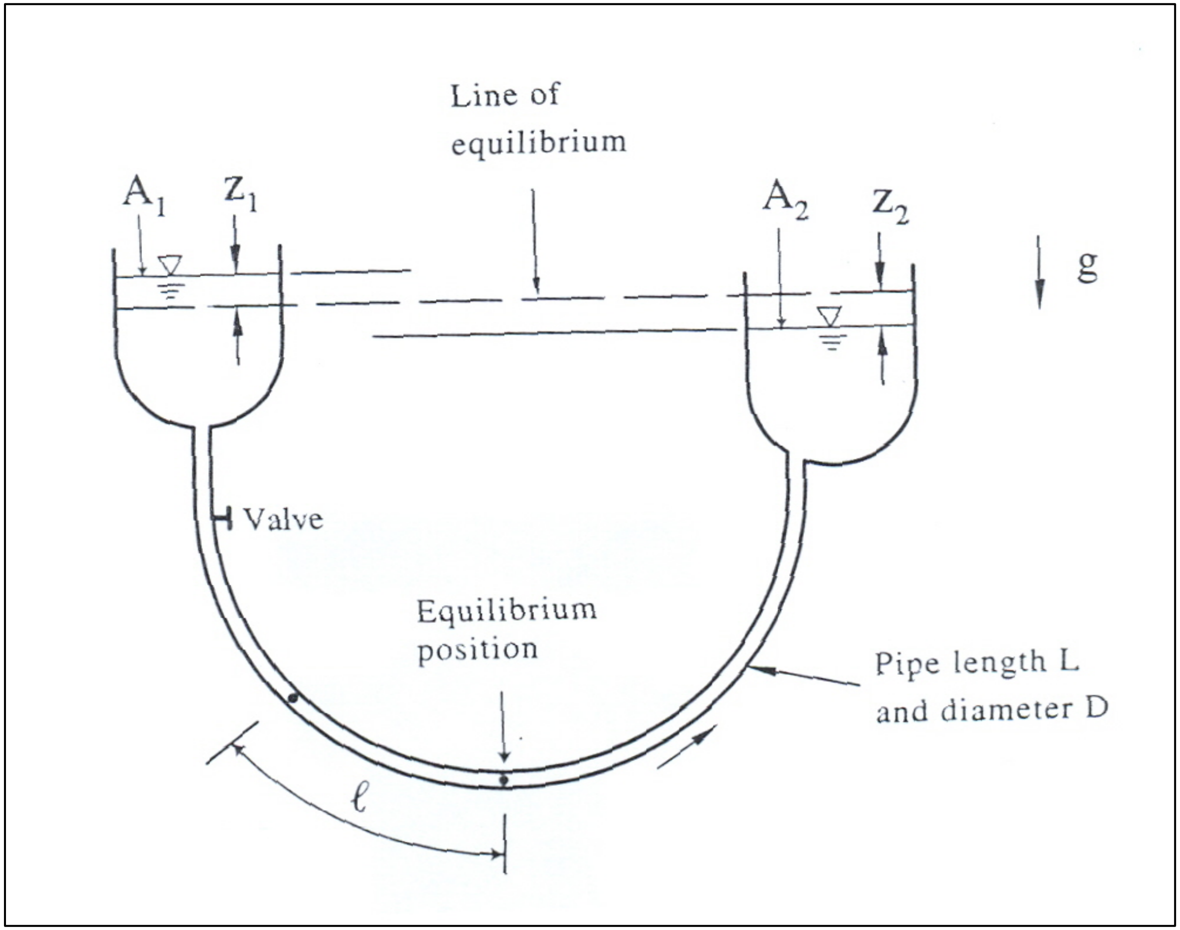
Fluid Acceleration Due to Valve Opening

$$t = \frac{LV_0}{2gH} \ell n \left(\frac{V_0 + V}{V_0 + V} \right)$$

$$t = 2.65 \frac{LV_0}{gH}$$

For practical purposes, to can be regarded as the time to establish steady flow velocity V_o

Flow Oscillations in Interconnected Tanks



Flow Oscillations in Interconnected Tanks

$$\frac{d^2\ell}{dt^2} \pm \frac{1}{2L}\left(\sum K + \frac{fL}{D}\right)\left(\frac{d\ell}{dt}\right)^2 + \frac{gA}{L}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)\ell = 0$$

- The sign in front of the second term in the above equation is minus for diminishing values of (d/dt < 0), and plus for increasing ((d/dt > 0)
- The equation can be solved numerically by using the initial conditions = o and d/dt = 0 at t = 0.
- Once the variation of with time is found, the variation of the water levels, z_1 and z_2 can be found

Unsteady Solution: Water Hammer

- Water hammer is the pressure wave created by sudden flow changes generated by rapid valve switching or unexpected pump shutdown such as encountered at power failure
- It is a rapidly varying unsteady flow (hydraulic transient) that cannot be treated as quasi-steady. It must be analyzed as truly unsteady flow.
- The pressure waves (surges) in water hammer often are of such high amplitudes that they cause damage to pipes, pumps, valves, and other fittings

Propagation of Small Pressure Disturbances in Pipes

- Pressure waves of small amplitudes (i.e., sudden pressure changes created by small disturbances) propagate in a pipe at the following celerity (i.e., wave speed in stationary fluid):

$$C = \sqrt{\frac{E}{\rho}}$$

Celerity of Water Hammer Waves

$$C = \frac{\sqrt{E/\rho}}{\sqrt{1+(E/E_p)(D/\delta)\varepsilon}}$$

- E_p is the Young's modulus of the pipe material:
- δ is the thickness of the pipe wall
- and ε is a dimensionless factor equal to 1.0 when the pipe wall is thin (i.e. when D/8 > 25)

Celerity of Water Hammer Waves

- For thick-walled pipes (D/8 < 25), the factor ε differs for different conditions as follows:
- Case 1: For a pipeline anchored at upstream,

$$\varepsilon = \frac{2\delta}{D}\left(1+\mu_p\right) + \frac{D}{(D+\delta)}\left(1-\frac{\mu_p}{2}\right)$$

- Case 2: For a pipeline anchored against longitudinal movement,

$$\varepsilon = \frac{2\delta}{D}\left(1+\mu_p\right) + \frac{D}{(D+\delta)}\left(1-\frac{\mu_p^2}{2}\right)$$

- Case 3: For a pipeline with expansion joints throughout its length,

$$\varepsilon = \frac{2\delta}{D}\left(1+\mu_p\right) + \frac{D}{(D+\delta)}$$

Rise and Drop of Pressure in Pipe Due to Sudden Valve Closure

- Assume the celerity of the water hammer to be C. and the mean velocity (discharge velocity) in the pipe before valve closure to be V, maximum rise of pressure in the pipe on the upstream side of the valve is :

$$\Delta P = \rho C V$$

- From the above equation. the rise of pressure head is

$$\Delta H = \frac{CV}{g}$$

Rise and Drop of Pressure in Pipe Due to Sudden Valve Closure

- maximum water pressure in the pipe upstream of the valve caused by instantaneous valve closure is

$$P_2 = P_s - \rho C V$$

- From the above equation. the rise of pressure head is

$$H_1 = H_s + \frac{CV}{g}$$

Rise and Drop of Pressure in Pipe Due to Sudden Valve Closure

- Immediately after valve closure, P_2 will be either of an amount ΔP below P_0, or be the same as P_v (vapor pressure), whichever is greater. Therefore

$$P_2 = P_0 - \rho CV \quad \text{(when } P_2 > P_v \text{)}$$

- Or

$$P_2 = P_v \quad \text{(when } P_2 \leq P_v \text{)}$$

Water Hammer Force on Valve

- The thrust (force) acting on a valve due to water hammer is

$$F_v = A(P_1 - P_2)$$

- For a valve in the middle of a pipe

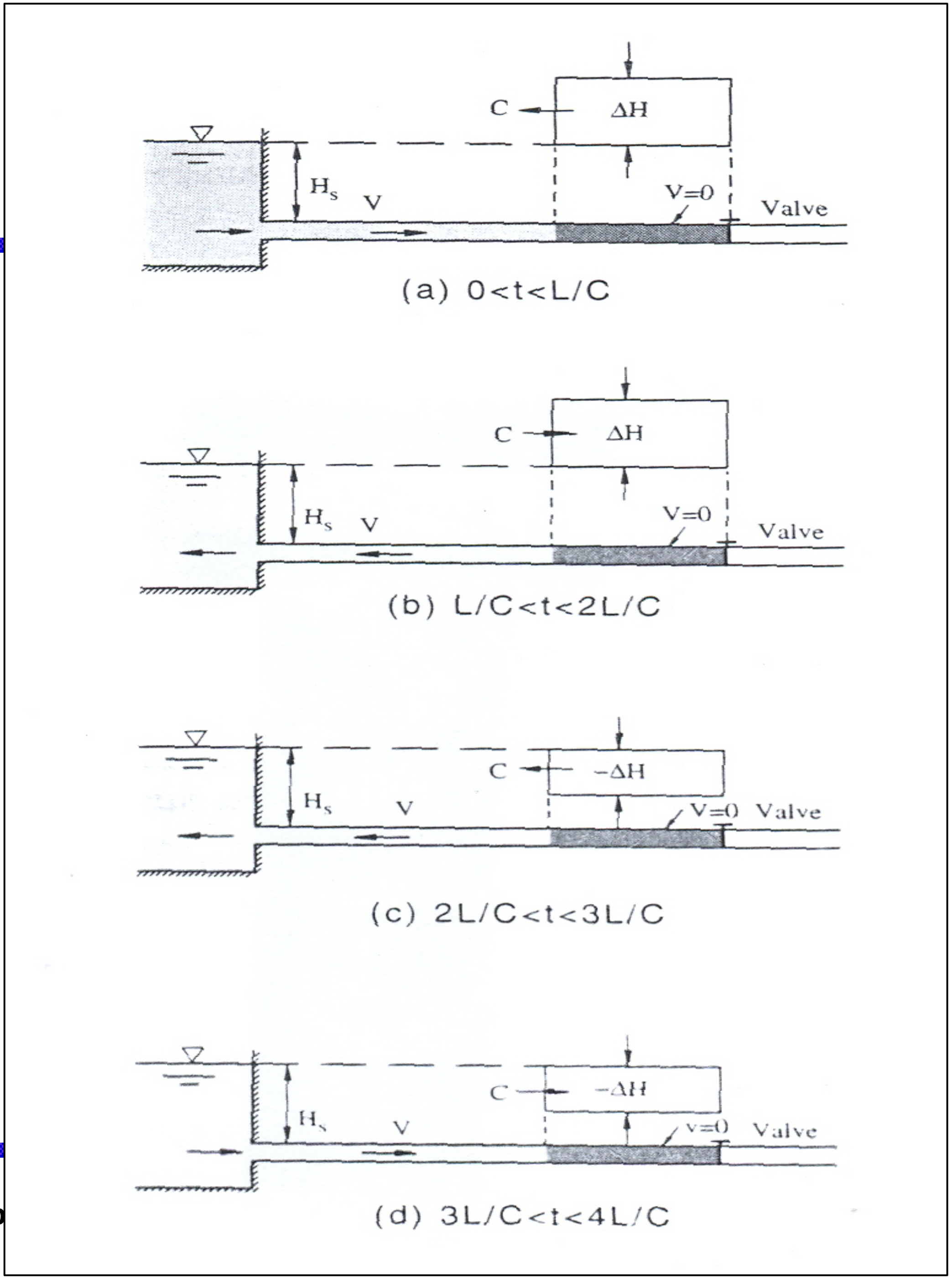
$$F_v = A(P_s - P_o + 2\rho VC) \quad \text{(when } P_2 > P_v\text{)}$$

$$F_v = A(P_s - P_o + \rho VC) \quad \text{(when } P_2 > P_v\text{)}$$

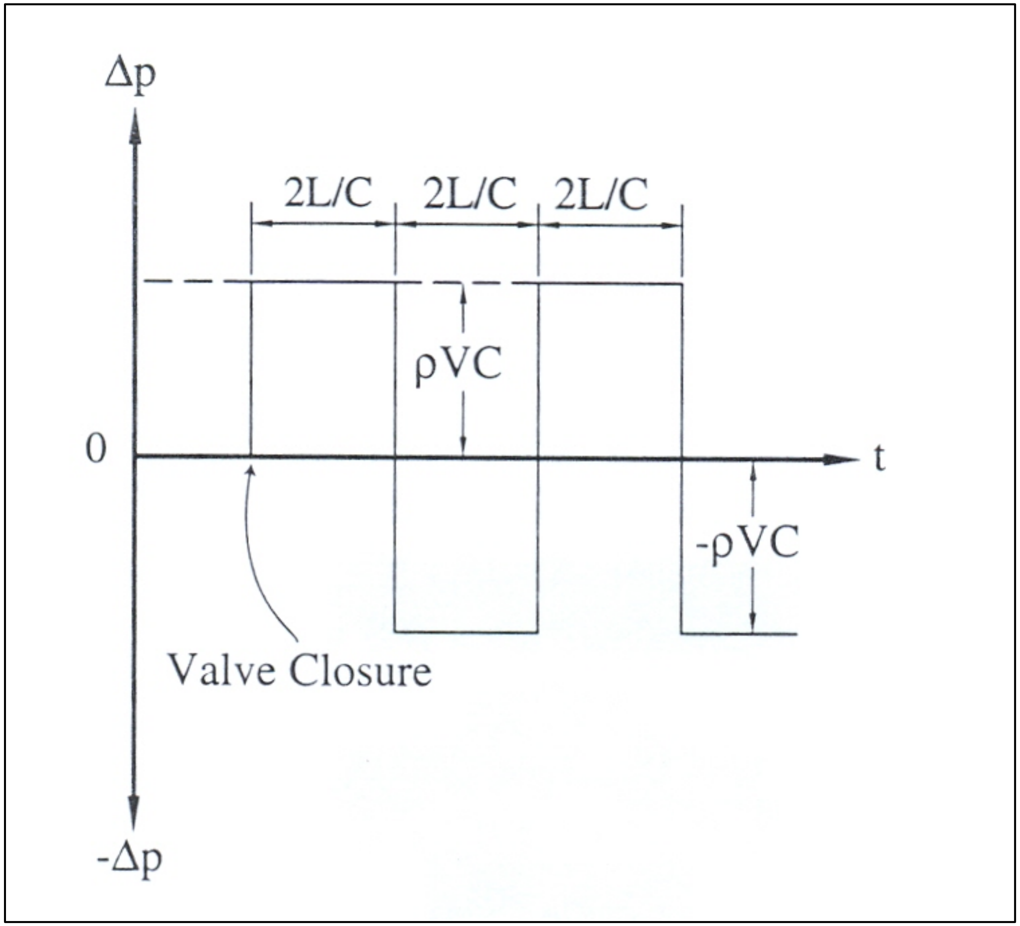
- For a valve at or near the pipe exit:

$$F_v = A(P_s - P_o + \rho VC)$$

Water Hammer Wave Propagation Due to Sudden Valve Closure



Variation of Pressure With Time at Valve Location Due to Sudden Valve Closure



Water Hammer Caused by Partial Closure of Valve

- A valve is closed partially such that the velocity in the pipe is decreased from V to V'
- The rise of water pressure head in the pipe due to the water hammer will be

$$\Delta H' = \frac{C\,(V - V')}{g}$$

Water Hammer with Finite Closure Time

- The distance X upstream from the valve

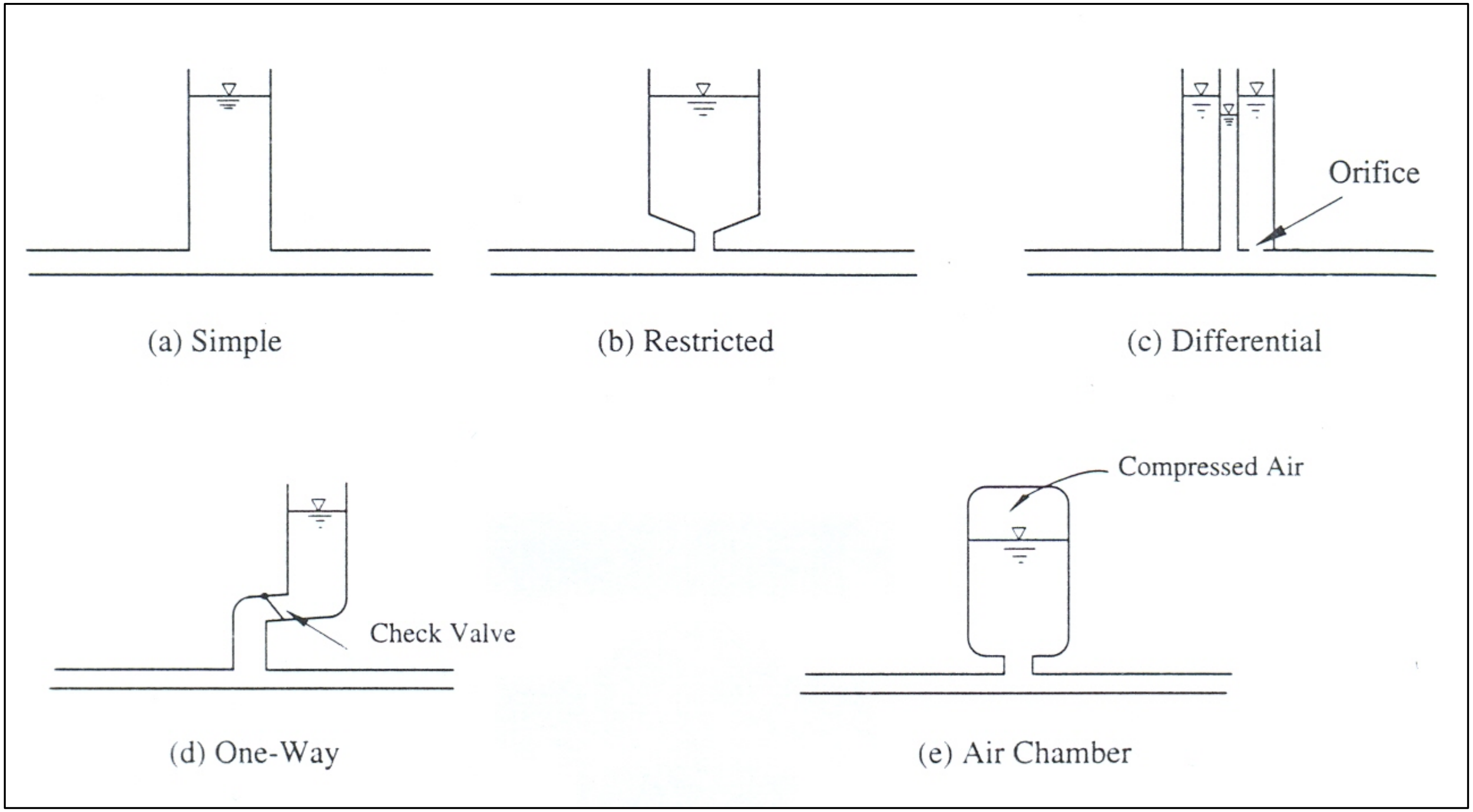
$$X = L - \frac{Ct_c}{2}$$

- Slow closure (t_c> 2L/C): For t_c > 2L/C (slow closure), the maximum pressure at the valve location is

$$\Delta P_s = \frac{2L/C}{t_c}\Delta P = \frac{2L\rho V}{t_c}$$

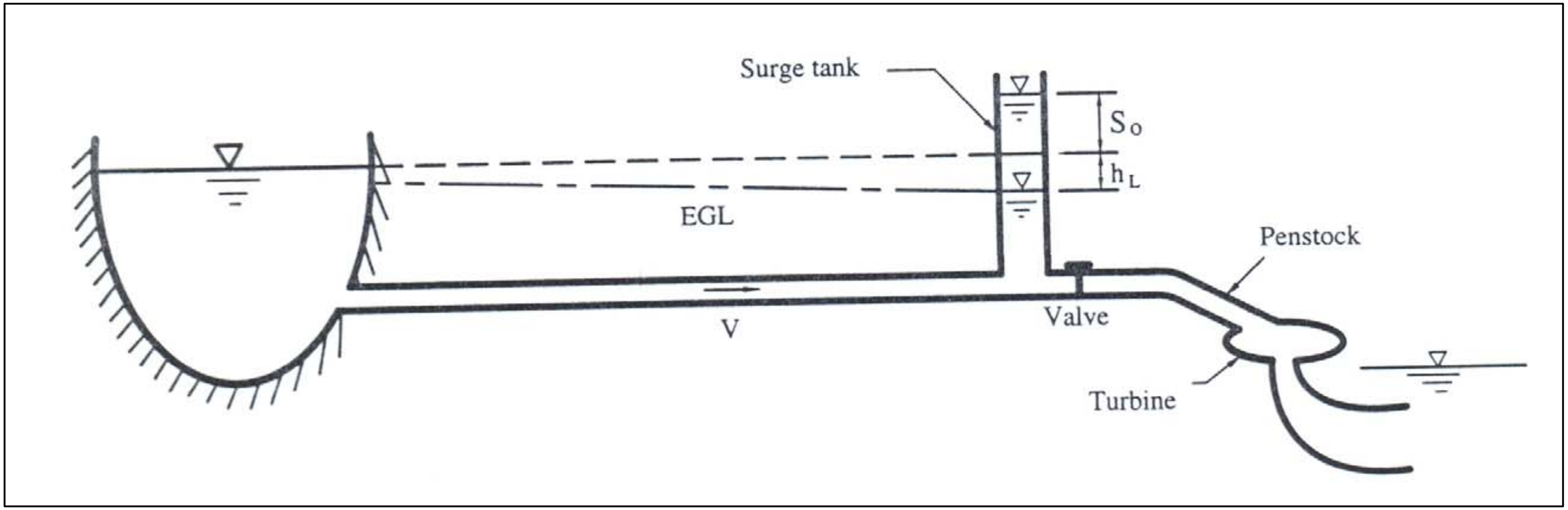
- The maximum pressure head generated from the full closure of a valve slowly (in time t_c > 2L/C) $\Delta H_s = \frac{2V_0 L}{g t_c}$

Surge Tanks



Various types of surge tanks

Surge Tanks



Maximum surge height generated by sudden valve closure

Surge Tanks

- S is the surge height, which is the height of water in the surge tank above its equilibrium value

$$\frac{d^2S}{dt^2} \pm \left(\frac{f}{2D}\frac{A_s}{A}\right)\left(\frac{dS}{dt}\right)^2 + \left(\frac{g}{L}\frac{A}{A_s}\right)S = 0$$

$$t_0 = 2\pi\sqrt{\frac{LA_s}{gA}}$$

$$S_0 = \sqrt{h_L^2 + \left(\frac{V_0 A}{\Phi A_s}\right)} = \sqrt{h_L^2 + \frac{ALV_o^2}{A_s g}}$$

- When h_L is neglected

$$S_0 = V_0\sqrt{\frac{LA}{gA_s}}$$

$$\Delta P_1 = \frac{\rho CV}{1 + \frac{CA_s}{C_t A}}$$